Recursion Theorems
Spring 2019, CS24, Dr. Ostheimer

Theorem 1 *Let* c_1 *and* c_2 *be real numbers. Suppose that* $r^2 - c_1 r - c_2 = 0$ *has two distinct roots* r_1 *and* r_2*. Then the sequence* $\{a_n\}$ *is a solution of the recurrence relation* $a_n = c_1 a_{n-1} + c_2 a_{n-2}$ *if and only if* $a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$ *for* $n = 0, 1, 2, \ldots,$ *where* α_1 *and* α_2 *are constants.*

Theorem 2 *Let* c_1 *and* c_2 *be numbers with* $c_2 \neq 0$*. Suppose that* $r^2 - c_1 r - c_2 = 0$ *has only one root* r_0*. A sequence* a_n *is a solution of the recurrence relation* $a_n = c_1 a_{n-1} + c_2 a_{n-2}$ *if and only if* $a_n = \alpha_1 r_0^n + \alpha_2 n r_0^n$*, for* $n = 0, 1, 2, \ldots,$ *where* α_1 *and* α_2 *are constants.*

Theorem 3 *Let* f *be an increasing function that satisfies the recurrence relation*

$$f(n) = af(n/b) + cn^d$$

whenever $n = b^k$*, where* k *is a positive integer,* $a \geq 1$*,* b *is an integer greater than* 1*, and* c *and* d *are real numbers with* c *positive and* d *nonnegative. Then*

- $f(n)$ *is* $O(n^d)$ *if* $a < b^d$*,*
- $f(n)$ *is* $O(n^d \log n)$ *if* $a = b^d$*, and*
- $f(n)$ *is* $O(n^{\log_b a})$ *if* $a > b^d$*.*