

Group work on divide and conquer
CS24, Dr. Ostheimer

A theorem from the text:

Theorem 1. *Let f be an increasing function that satisfies the recurrence relation*

$$f(n) = af(n/b) + cn^d$$

whenever $n = b^k$, where k is a positive integer, $a \geq 1$, b is an integer greater than 1, and c and d are real numbers with c positive and d nonnegative. Then

- $f(n)$ is $O(n^d)$ if $a < b^d$,
- $f(n)$ is $O(n^d \log n)$ if $a = b^d$, and
- $f(n)$ is $O(n^{\log_b a})$ if $a > b^d$.

- (1) Let $t(n) = t(n/2) + 1$. Matching the recurrence relation for t to the theorem above, what are a, b, d ?
- (2) Which case of the theorem are we in?
- (3) What does the theorem predict for the big-O estimate for t ?
- (4) Use guess and check to find a closed formula for $t(n)$ if $t(1) = 17$.
- (5) Find a simple big-O estimate for your closed formula.
- (6) Did the theorem give a good prediction?