CS 024, Discrete Structures II
Induction Practice

Recall that **N** is the set of non-negative integers.

Theorem. Let $f(n)$ be the function from **N** to **N** defined as follows:

- $f(0) = 1$;
- $f(n) = f(n-1) + 2n - 1$ if $n > 0$.

Then $f(n) = n^2 + 1$ for all $n \in N$.

1. Re-order the following statements to make a valid proof. Your answer to this question must be an ordered list of letters.

   (a) Therefore $f(n) = n^2 + 1$.

   (b) For the inductive step, we assume that $P(k)$ holds for all $k < n$, and we must prove that $P(n)$ holds.

   (c) We are given that $f(n) = f(n-1) + 2n - 1$.

   (d) Therefore $f(n) = (n-1)^2 + 1 + 2n - 1$.

   (e) Since $n - 1 < n$, we know that $f(n-1) = (n-1)^2 + 1$.

   (f) For the basis step, we must prove that $P(0)$ is true.

   (g) $(n-1)^2 + 1 + 2n - 1 = (n^2 - 2n + 1) + 1 + 2n - 1 = n^2 + 1$.

   (h) Let $P(n)$ be the statement "$f(n) = n^2 + 1$".

   (i) This completes the proof.

   (j) Since $f(0)$ is defined to be 1, this is obvious.

   (k) In other words, we assume that $f(k) = k^2 + 1$ for all $k < n$, and we must prove that $f(n) = n^2 + 1$.

   (l) In other words, we must prove that $f(0) = 0^2 + 1$.

   Your list of letters here:

2. Classify each statements according to whether they are assumptions, definitions, statements of intent or deductions. Write the classification to the left of each statement on the previous page.

3. In which statement is the inductive assumption used? Write the appropriate letter here:

4. True or false: $P(n) = n^2 + 1$.