Graph Isomorphism – Group Work – Part I
CS24, Dr. Ostheimer

Learning objectives:

- to work on your close reading skills;
- to begin to develop both a precise and an intuitive understanding of graph isomorphism;
- to prepare you to look at The Graph Isomorphism Problem and its place in the $\mathcal{P}$ versus $\mathcal{NP}$ problem.

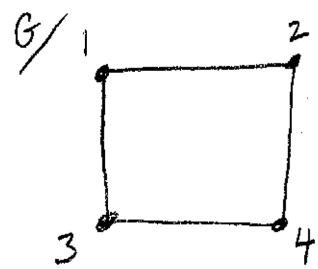
Definition: Let $G = (U, E)$ and $H = (V, F)$ be two simple, undirected graphs. Let $f : U \to V$. f is an *isomorphism from G to H* if the following all hold:

- f is a one-to-one correspondence from U to V; and
- for all $u_1, u_2 \in U$, $\{u_1, u_2\} \in E$ if and only if $\{f(u_1), f(u_2)\} \in F$.

Your questions:

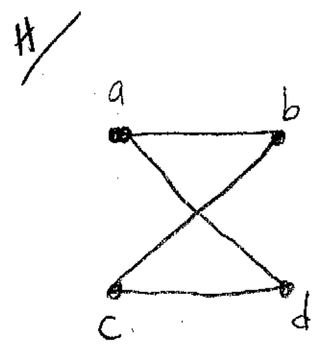
1. Read the first definition of *isomorphism* out loud as a group at least two times.
2. List the terms you need to understand in order to understand this definition.
3. Find the definition of *simple, undirected graph* from class. Read it out loud as a group at least two times.
4. Going back to the definition of *isomorphism*, what are the elements of U and V called?
5. What are the elements of E and F called?
6. Notice all the times that round brackets are used in the definition, and all the times that curly brackets are used. Each time, try switching the bracket type and make sure you see why that wouldn't make sense.
7. Find the definition of *one-to-one correspondence* and read it out loud as a group at least twice.
8. Are there any terms in the definition of *one-to-one correspondence* that you need to understand? You know what to do!

9. Here is an example of a simple, undirected graph $G = (U, E)$. What is U and what is E in this example?



10. In your answer to the last question, was E a set of two-element subsets of U? If not: oops. Try again.

11. Here is an example of a simple, undirected graph $H = (V, F)$. What are V and F in this example?



12. True or false: $U = V$.

13. True or false: $G = H$.

14. Let $f : U \to V$ be given by $f(1) = a, f(2) = b, f(3) = c, f(4) = d$.

    (a) True or false: f is a one-to-one correspondence from U to V.

    (b) Find an edge $\{u_1, u_2\} \in E$ such that $\{f(u_1), f(u_2)\} \in F$, or state that there is no such edge.

    (c) Find an edge $\{u_1, u_2\} \in E$ such that $\{f(u_1), f(u_2)\} \notin F$, or state that there is no such edge.

    (d) True or false: f is an isomorphism from G to H.

15. Let $f : U \to V$ be given by $f(1) = a, f(2) = b, f(3) = d, f(4) = c$.

(a) True or false: f is a one-to-one correspondence from U to V.

(b) Find an edge $\{u_1, u_2\} \in E$ such that $\{f(u_1), f(u_2)\} \in F$, or state that there is no such edge.

(c) Find an edge $\{u_1, u_2\} \in E$ such that $\{f(u_1), f(u_2)\} \notin F$, or state that there is no such edge.

(d) True or false: f is an isomorphism from G to H.