CS 024, Discrete Structures II
Recursion and Counting

1. For $r, n \in \mathbf{N}$ with $r \leq n$, let $P(n, r)$ be the number of r-permutations of a set with n elements. An r-*permutation* of a set is an ordering of r of the elements of the set.

   (a) Find a recursive formula for $P(n, r)$.

   (b) Find a non-recursive formula for $P(n, r)$ that uses the factorial symbol.

   (c) Find a non-recursive formula for $P(n, n)$.

2. For $r, n \in \mathbf{N}$ with $r \leq n$, let $C(n, r)$ be the number of r-element subsets of a set with n elements.

   (a) Find a recursive formula for $C(n, r)$.

   (b) Find a non-recursive formula for $C(n, r)$ that uses the factorial symbol.

   (c) Find a formula relating $P(n, r)$ and $C(n, r)$.

3. Go back over what you have done and look at where you used mulitplication and where you used addition. Relate this to the sum and product rules given here:

   - **Sum Rule.** Let A and B be disjoint sets. Then $|A \cup B = |A| + |B|$.
   - **Product Rule.** Let A and B be (arbitrary) sets. Then $|A \times B = |A||B|$.