Group Work
CSC-024, Prof. Ostheimer

Some of the questions below refer to this definition:

Definition 1 *Let $f, g : \mathbf{N} \to [0, \infty)$. f is* friendly toward g *if and only if there exist constants C and k such that*

$$f(x) \leq Cg(x)$$

for all $x \geq k$. C and k are called witnesses.

1. True or false: there exists an $x \in \mathbb{N}$ such that $3x^2 + 17 \leq x^2$.
2. True or false: there exists a constant k such that $3x^2 + 17 \leq x^2$ whenever $x \geq k$. If your answer to this is "true", find a k that works.
3. True or false: $3x^2 + 17 \leq 15x^2$ for all $x \in \mathbb{N}$.
4. True or false: $3x^2 + 17 \leq 15x^2$ for all $x \geq 10$.
5. True or false: There exists a constant k such that $3x^2 + 17 \leq 15x^2$ for all $x \geq k$. If your answer is "true": find a k that works.
6. Read the definition of *friendly* out loud.
7. True or false: $3x^2 + 17$ is friendly toward x^2.
8. True or false: There exists a constant k such that $3x^2 + 17 \leq 4x^2$ for all $x \geq k$. If your answer is "true": find such a k.
9. Prove that $3x^2 + 17$ is friendly toward x^2.