Group Work
CSC-024, Prof. Ostheimer

Several of the questions below refer to this definition (and the rest do not!):

Definition 1 *Let $f, g : \mathbf{N} \to [0, \infty)$. f is* friendly toward g *if and only if there exist constants C and k such that*

$$f(x) \leq Cg(x)$$

for all $x \geq k$. C and k are called witnesses *to the fact that f is friendly toward g.*

1. True or false: there exists an $x \in \mathbb{N}$ such that $3x^2 + 17 \leq x^2$.
2. True or false: there exists a constant k such that $3x^2 + 17 \leq x^2$ for all $x \geq k$. If your answer to this is "true", find a k that works.
3. True or false: $3x^2 + 17 \leq 15x^2$ for all $x \in \mathbb{N}$.
4. True or false: $3x^2 + 17 \leq 15x^2$ for all $x \geq 10$.
5. True or false: There exists a constant k such that $3x^2 + 17 \leq 15x^2$ for all $x \geq k$. If your answer is "true": find a k that works.
6. Read the definition of *friendly* out loud.
7. Many students ask me this: "What's the difference between x and k in the definition"? How would you answer that question?
8. True or false: $3x^2 + 17$ is friendly toward x^2.
9. True or false: There exists a constant k such that $3x^2 + 17 \leq 4x^2$ for all $x \geq k$. If your answer is "true": find such a k.
10. Prove that $3x^2 + 17$ is friendly toward x^2.
11. For which of the questions above did you actually need to know the definition of "friendly" in order to answer them?
12. Why do you think C and k are called "witnesses"?
13. Some of you have already been exposed to this concept of "friendly" in your other classes. Have you? If so, what's the real name of the concept being defined here?