Proving that a given language is not regular
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How do we go about proving a theorem like this:
Thm. *BLAH-BLAH* is not regular.

Outline of proof.
Let L be the language *BLAH-BLAH*.
Suppose that L is regular.
We will demonstrate that this leads to a logical contradiction.

By Kleene's Theorem, there exists a finite automaton A that accepts L.
Let N be the number of states in A.
Let $w =$?.
How do we, the theorem-prover, choose w?
This is the hardest part of the proof.
We need to choose w such that

- $w \in L$, and

- we are guaranteed to find a loop in the accepting path for w, which, when pumped, yields a word u which is not in L.

Notice that u is certainly accepted by A: its path ends in the same place as w's path, and w is accepted by A since it's in L.

Let p be the accepting path for w.
Notice that p starts at the initial state of A and ends in a final state of A.
Let r be the label of the start of path p leading from the initial state up to the start of the loop we want to pump,
let s be the label of the loop,
and let t be the label of the rest of p.
Then $w = rst$.
Furthermore, by our choice of w we can infer that $r =$?, $s =$? and $t =$?.

Let $k =$? and let $u = rs^k t$.
We will now show that we have chosen k such that $u \notin L$...
Notice that u is accepted by A since its path has the same end state as w, so its path ends in a final state.
However, $u \notin L$ because ...

We have reached a contradiction, so we can deduce that our assumption that L is regular is false. $\bullet$