Summary of finite automata
CS 161, Dr. Ostheimer

Definition 1. *Let Σ be an alphabet. A* finite automaton over Σ *is a collection of* 2 *things:*

(1) *a finite set S of states including*
   - **exactly one** *initial state*
   - *zero or more final states*

(2) *for each state $s \in S$ and for each $\sigma \in \Sigma$, exactly one directed edge leaving s labeled σ.*

Definition 2. *Let A be a finite automaton over Σ. The* language accepted by A *is the set of words $w \in \Sigma^*$ such that the path starting at the initial state and labeled w ends at a final state.*

Remark 1. *Note that by the definition of finite automaton, for a given word $w \in \Sigma^*$, there is one and only one path starting at the initial state labeled w; therefore, the previous definition makes sense.*

Definition 3. *Let Σ be an alphabet. A* generalized transition graph over Σ *is a collection of* 2 *things:*

(1) *a finite set S of states including*
   - **zero or more** *initial states*
   - *zero or more final states*

(2) **for each pair $s, t \in S$, zero or more directed edges, each labeled with a regular expression over Σ.**

Definition 4. *Let G be a generalized transition graph over Σ. The* language accepted by G *is the set of words $w \in \Sigma^*$ such that* **there exists** *a path*

- *starting at an initial state and ending at a final state, that is*
- *labeled with a regular expression r such that w is in the language defined by r.*

Remark 2. *Note for a generalized transition graph G over Σ and a word $w \in \Sigma^*$, many things can happen, including*

(1) *there might not be any paths labeled r such that $w \in$ language(r) from an initial state;*

(2) *there might be many paths labeled r such that $w \in$ language(r) from an initial state to a final state and at the same time there might be many paths labeled r such that $w \in$ language(r) from an initial state to a non-final state;*

This is a source of what we call nondeterminism *– we may have choices about how to trace w.*

Definition 5. *A generalized transition graph G over Σ is also called a* transition graph *if all of the edge labels are words Σ^*.*