

Pumping Proof
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Let L be the language of words w over $\{a, b\}$ of the form $w = a^n b^n$, where $n = 0, 1, 2, \dots$

1. Give three examples of words in L .
2. Give three examples of words in the language defined by a^*b^* that are not in L .
3. Give three examples of words in the language EQUAL that are not in L .
4. Re-order the following sentences to create a proof of the fact that there does not exist a finite automaton with 7 states that accepts L .
 - (a) Let s be the state that we end at when we trace w (starting at the initial state).
 - (b) Since w is in L , w is accepted by A .
 - (c) Now consider the word $w' = xy^2z$.
 - (d) We have reached a contradiction.
 - (e) Therefore, s is a final state.
 - (f) Since a^8 has length 8 and A has only 7 states, when we trace a^8 (starting at the initial state), we must hit at least one state twice. Let t be the first state that is hit twice.
 - (g) Suppose A is a finite automaton with 7 states that accepts L .
 - (h) It follows that $x = a^i$ for some nonnegative integer i , $y = a^j$ for some positive integer j , $z = a^k b^8$ for some nonnegative integer k and $i + j + k = 8$.
 - (i) When we trace w' , x takes us from the initial state to state t , y takes back to state t , the second y takes us back to state t again, and then z takes us to state s .
 - (j) Since $j > 0$, $w' \notin L$.
 - (k) Let x , y and z be words defined as follows: $w = xyz$; x is the word we get when tracing w and stopping the first time we hit state t ; y is the word we get when continuing from there in our trace of w and stopping the next time we hit state t ; z is the word we get when continuing from there and finishing the trace of w .
 - (l) Since s is final, this implies that w' is accepted.
 - (m) Therefore, there is no finite automaton with 7 states that accepts L .
 - (n) Let $w = a^8 b^8$.
 - (o) $w' = a^i a^j a^j a^k b^8 = a^8 a^j b^8$.
5. Modify the proof above as little as possible to prove that L is not regular.