Recursive Definitions, CSC-161, Dr. Ostheimer

(1) Let L be a language over X, We define $S(L)$ as follows;
   - $\Lambda \in S(L)$;
   - if $u \in S(L)$ and $v \in L$, then $uv \in L$.

   (a) Why is this called a recursive definition?
   (b) True or False: $L \subseteq S(L)$. (Justify your answer.)
   (c) True or False: $S(L) \subseteq L$. (Justify your answer.)
   (d) True or False: $S(L) \subseteq X^*$. (Justify your answer.)
   (e) We already gave $S(L)$ a different name. What is that name?
   (f) Which do you like better, the definition we had last week in class, or, rather, this one?

(2) How could you modify the definition above to define the positive Kleene closure of L?

(3) Let X be an alphabet. Create a recursive definition for X^*.

(4) Let X be an alphabet. Let $T(X)$ be defined as follows
   - $\Lambda \in T(X)$;
   - For all $x \in X$, $x \in T(X)$;
   - For all $w \in T(X)$ and all $x \in X$, $xwx \in T(X)$.

   (a) Why is this called a recursive definition?
   (b) True or False: $X \subseteq T(X)$. (Justify your answer.)
   (c) True or False: $T(X) \subseteq X$. (Justify your answer.)
   (d) True or False: $T(X) \subseteq X^*$. (Justify your answer.)
   (e) We already gave $T(X)$ a different name. What is that name?
   (f) Which do you like better, the definition we had last week in class, or, rather, this one?

(5) Let $X = \{a, b\}$, $Z = \{\mathbf{\Lambda}, \mathbf{a}, \mathbf{b}\}$ and $Y = \{(,), +, *\}$.
Let $U(X)$ be the language over $Z \cup Y$ defined as follows:

- For all $x \in Z$, $x \in U(X)$;
- if $w \in U(X)$, then $(w) \in U(X)$ and $w^* \in U(X)$;
- if $u, v \in U(X)$, then $uv \in U(X)$ and $u + v \in U(X)$.

(a) What is the cardinality of the alphabet $Z \cup Y$?
(b) Come up with 5 interesting words w over $Z \cup Y$ such that $w \in U(X)$.
(c) Come up with 5 interesting words w over $Z \cup Y$ such that $w \notin U(X)$.
(d) True or False: $Z \subseteq U(X)$. (Justify your answer.)
(e) True or False: $U(X) \subseteq Z$. (Justify your answer.)
(f) True or False: $U(X) \subseteq X^*$. (Justify your answer.)
(g) True or False: $U(X) \subseteq (Z \cup Y)^*$. (Justify your answer.)
(h) Read the definition at the bottom of page 35 in Cohen. So: what's the normal term for $U(X)$?
(i) Cohen emphasizes a versus $\mathbf{a}$. What is the point of this?
(j) What is the difference between Λ and $\mathbf{\Lambda}$? What is the length of the word Λ? What is the length of the word $\mathbf{\Lambda}$?
(k) How would you modify this definition to define the set of regular expressions over the alphabet $\{x, y, z\}$ instead of over $\{a, b\}$, as we have done here?