Excerpts from an Old Exam CSC-161, Dr. Ostheimer

1. **Closure and Regularity:**

   Assume for this problem that L and M are languages over the alphabet $X = \{a, b\}$. For the following True-False questions, if your answer is false, produce languages L and M that prove it.

   (a) True or false: If L is finite, then L is regular.

   (b) True or false: If L is regular and M is regular. then $L \cup M$ is regular.

   (c) True or false: If L is regular and $L \cup M$ is not regular, then M is not regular.

   (d) True or false: If $L \cup M$ is regular, then at least one of L and M is regular.

2. **Using Closure in Non-regularity Proofs:** Let $X = \{a, b\}$. In class we have proven that PALINDROME(X) is not regular using pumping. For this problem, you may assume this is true. Let L be the set of all words in X^* of length 4 or less.

   (a) True or false: L is finite.

   (b) True or false: L is regular.

   (c) Write a short proof that PALINDROME(X)$-L$ is not regular. Recall that for sets A, B, $A-B$ is the set of those elements of A that are not elements of B. Do *not* use pumping, but rather use what you know about closure. (Three or four complete sentences should suffice.)

3. **Understanding Pumping:** Answer the following True/False questions. If your answer is "True", provide an example.

   (a) True or false: There exists a finite automaton A with 5 states over X such that language$(A) = \{a^i b^{3i} a^i \text{ s.t.} i = 0, 1, 2, 3, \ldots\}$.

   (b) True or false: There exists a finite automaton A with 5 states over X such that language$(A) \supseteq \{a^i b^{3i} a^i \text{ s.t.} i = 0, 1, 2, 3, \ldots\}$.

   (c) True or false: There exists a finite automaton A with 5 states over X such that language$(A) \subseteq \{a^i b^{3i} a^i \text{ s.t.} i = 0, 1, 2, 3, \ldots\}$.

   (d) True or false: There exists a finite automaton A with 5 states over X such that A accepts $aabbbbbbaa$ and language$(A) \subseteq \{a^i b^{3i} a^i \text{ s.t.} i = 0, 1, 2, 3, \ldots\}$.