

Tentative Homework Problems CS 161

Languages

Cohen Chapter 2: 1, 2, 5, 6
Linz Chapter 1: 1, 3, 5, 6, 7, 8, 11

Definitions:

alphabet; language; word
Kleene closure (*), positive Kleene closure (+)
Kleene closed
product language
reverse, PALINDROME(X)

*From Linz, Formal Languages and Automata,
6th edition, 2017, Chapter 1*



1. How many substrings aab are in ww^Rw , where $w = aabbab$?
2. Use induction on n to show that $|u^n| = n|u|$ for all strings u and all n .
3. The reverse of a string, introduced informally above, can be defined more precisely by the recursive rules

$$\begin{aligned}a^R &= a, \\(wa)^R &= aw^R,\end{aligned}$$

for all $a \in \Sigma$, $w \in \Sigma^*$. Use this to prove that

$$(uv)^R = v^R u^R,$$

for all $u, v \in \Sigma^+$.

4. Prove that $(w^R)^R = w$ for all $w \in \Sigma^*$.
5. Let $L = \{ab, aa, baa\}$. Which of the following strings are in L^* : $abaabaaabaa$, $aaaabaaaa$, $baaaaabaaaab$, $baaaaabaa$? Which strings are in L^4 ?
6. Let $\Sigma = \{a, b\}$ and $L = \{aa, bb\}$. Use set notation to describe \bar{L} .
7. Let L be any language on a nonempty alphabet. Show that L and \bar{L} cannot both be finite.
8. Are there languages for which $\bar{L}^* = (\bar{L})^*$?
9. Prove that

$$(L_1 L_2)^R = L_2^R L_1^R$$

for all languages L_1 and L_2 .

10. Show that $(L^*)^* = L^*$ for all languages.
11. Prove or disprove the following claims.
 - (a) $(L_1 \cup L_2)^R = L_1^R \cup L_2^R$ for all languages L_1 and L_2 .
 - (b) $(L^R)^* = (L^*)^R$ for all languages L .