Functions (Section 2.3) Part I
CS-14, Dr. Ostheimer

1. Learning Objectives

- to get to know each other and start setting up study partners;
- to practice reading dense mathematic texts;
- to practice working with the logical notation taught in CS-14;
- to get ready to learn the concepts in Section 2.3.

2. Part 1

Recall from CS 14 that if A and B are sets, then $A \times B$, the Cartesian product of A and B, is the set of all ordered pairs (m, n) such that $m \in A$ and $n \in B$. Here's a definition which is probably new to you:

Definition 1. *A* relation *from set A to set B is a subset of $A \times B$.*

To practice with this definition, answer these questions:

(1) Give an example of a relation of cardinality 5 from $\{1, 2, 3\}$ to $\{2, 3, 4, 5\}$.
(2) What is the largest relation from $\{1, 2, 3\}$ to $\{2, 3, 4, 5\}$?
(3) How many relations are there from $\{1, 2, 3\}$ to $\{2, 3, 4, 5\}$?

3. Part 2

Here are 5 sample relations:

- $A = \{1, 2, 3\}$; $B = \{5, 6\}$; $R_1 = \{(1, 5), (1, 6), (2, 5), (3, 5)\}$.
- $A = \{1, 2, 3\}$; $B = \{5, 6\}$; $R_2 = \{(1, 5), (2, 5), (3, 6)\}$.
- $A = \{1, 2, 3\}$; $B = \{5, 6, 7, 8\}$; $R_3 = \{(1, 5), (2, 6), (3, 7)\}$.
- $A = \{1, 2, 3\}$; $B = \{5, 6, 7\}$; $R_4 = \{(1, 5), (1, 7), (2, 5), (3, 6)\}$.
- $A = \{1, 2, 3\}$; $B = \{5, 6, 7\}$; $R_5 = \{(1, 5), (2, 6)\}$.

Here are 4 pretend definitions:

- A relation R from A to B is *tall* if
$$\forall a \in A, \exists b \in B \mid (a, b) \in R.$$
- A relation R from A to B is *dark* if
$$\forall b \in B, \exists a \in A \mid (a, b) \in R.$$
- A relation R from A to B is *handsome* if
$$\forall a_1, a_2 \in A, b \in B, ((a_1, b) \in R \wedge (a_2, b) \in R) \rightarrow a_1 = a_2.$$
- A relation R from A to B is *charismatic* if
$$\forall a \in A, b_1, b_2 \in B, ((a, b_1) \in R \wedge (a, b_2) \in R) \rightarrow b_1 = b_2.$$

Here are your $20 = 5 \times 4$ questions: For each of the 5 relations above, decide whether or not it is tall, dark, handsome or charismatic. Note that a given relation might have none of these characteristics, some of them or all of them.