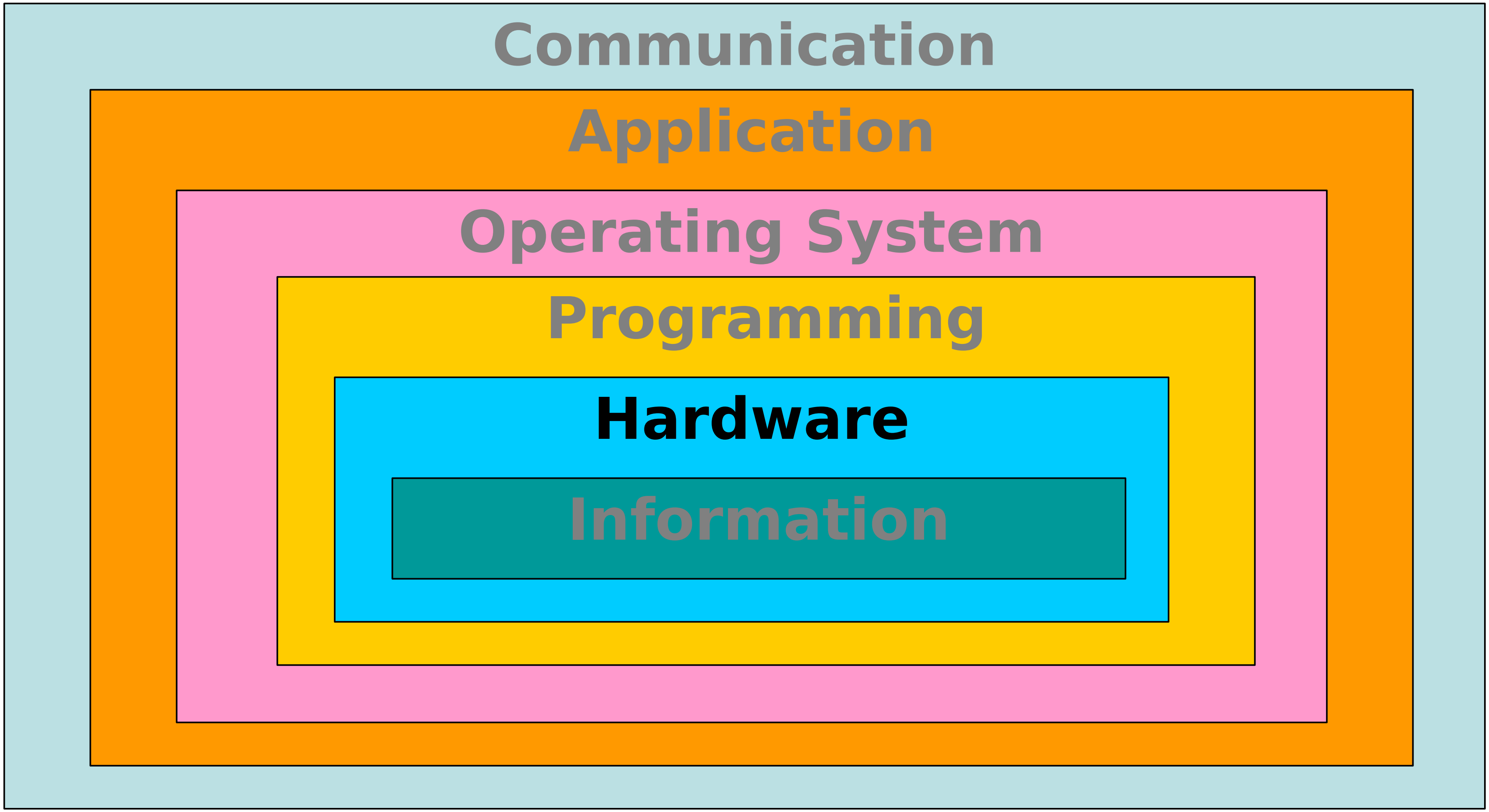
Chapter 4

Gates and Circuits

Layers of a Computing System



Chapter Goals

- Compare and contrast a half adder and a full adder
- Describe how a multiplexer works
- Explain how an S-R latch operates
- Describe the characteristics of the four generations of integrated circuits

Gates

- Let's examine the processing of the following six types of gates
  - NOT
  - AND
  - OR
  - XOR
  - NAND
  - NOR
- Typically, logic diagrams are black and white, and the gates are distinguished only by their shape

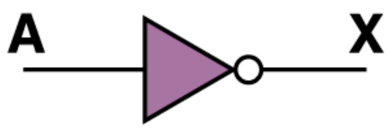
NOT Gate

- A **NOT** gate accepts one input value and produces one output value

Boolean Expression

$X = A'$

Logic Diagram Symbol



Truth Table

A	X
0	1
1	0

Figure 4.1 Various representations of a NOT gate

NOT Gate

- By definition, if the input value for a NOT gate is 0, the output value is 1, and if the input value is 1, the output is 0
- A NOT gate is sometimes referred to as an *inverter* because it inverts the input value

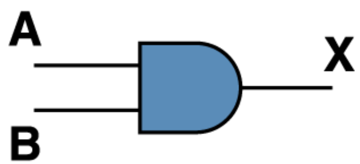
AND Gate

- An **AND** gate accepts two input signals
- If the two input values for an AND gate are both 1, the output is 1; otherwise, the output is 0

Boolean Expression

$$X = A \cdot B$$

Logic Diagram Symbol



Truth Table

A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

Figure 4.2 Various representations of an AND gate

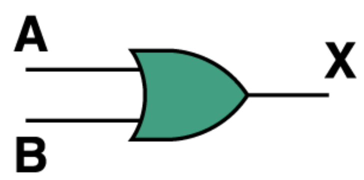
OR Gate

- If the two input values are both 0, the output value is 0; otherwise, the output is 1

Boolean Expression

$$X = A + B$$

Logic Diagram Symbol



Truth Table

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

Figure 4.3 Various representations of a OR gate

XOR Gate

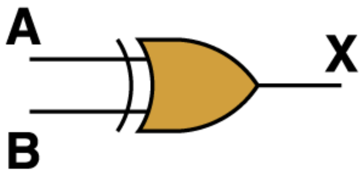
- **XOR**, or *exclusive* OR, gate
  - An XOR gate produces 0 if its two inputs are the same, and a 1 otherwise
  - Note the difference between the XOR gate and the OR gate; they differ only in one input situation
  - When both input signals are 1, the OR gate produces a 1 and the XOR produces a 0

XOR Gate

Boolean Expression

$$X = A \oplus B$$

Logic Diagram Symbol



Truth Table

A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

Figure 4.4 Various representations of an XOR gate

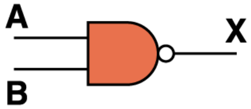
NAND and NOR Gates

- The **NAND** and **NOR** gates are essentially the opposite of the AND and OR gates, respectively

Boolean Expression

$X = (A \cdot B)'$

Logic Diagram Symbol



Truth Table

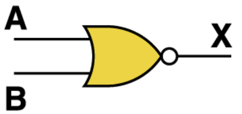
A	B	X
0	0	1
0	1	1
1	0	1
1	1	0

Figure 4.5 **Various representations of a NAND gate**

Boolean Expression

$X = (A + B)'$

Logic Diagram Symbol



Truth Table

A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

Figure 4.6 **Various representations of a NOR gate**

Review of Gate Processing

- A **NOT** gate inverts its single input value
- An **AND** gate produces 1 if both input values are 1
- An **OR** gate produces 1 if one or the other or both input values are 1

Review of Gate Processing

- An **XOR** gate produces 1 if one or the other (but not both) input values are 1
- A **NAND** gate produces the opposite results of an AND gate
- A **NOR** gate produces the opposite results of an OR gate

Homework

- **Read Chapter Four, Sections 4.1 – 4.3**
- **Exercise: P117, 18-29**

Constructing Gates

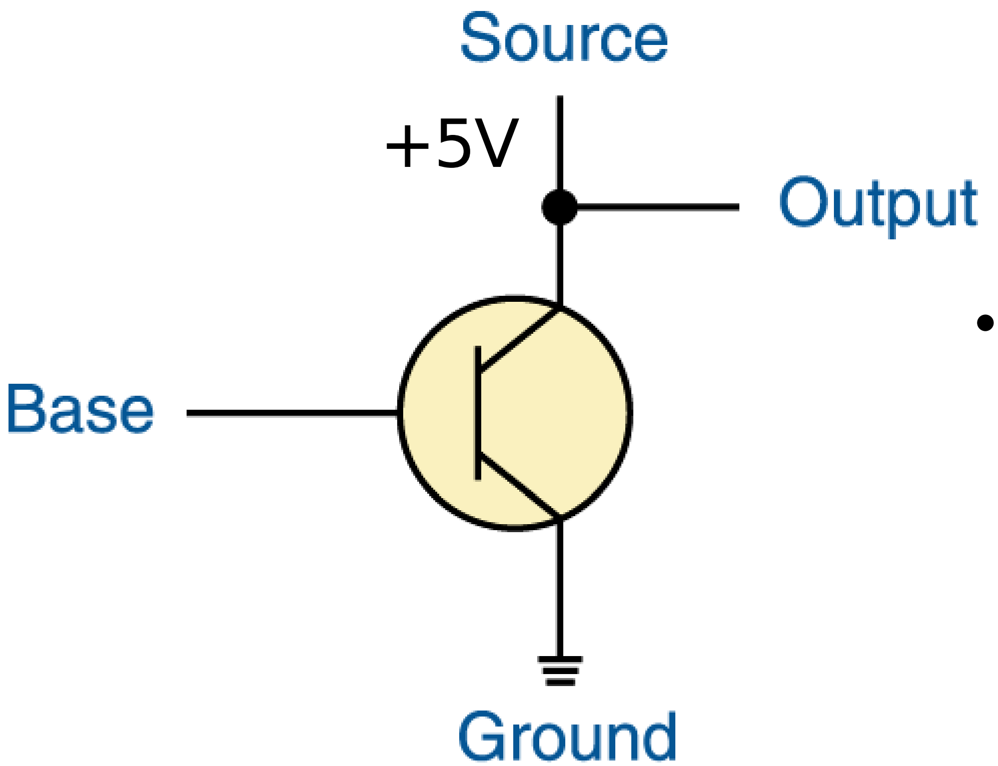


Figure 4.8 **The connections of a transistor**

- A transistor has three terminals
  - A **source**
  - A **base**
  - An **emitter**, typically connected to a ground wire
- If the electrical signal is grounded (base is high), it is allowed to flow through an alternative route to the ground (literally) where it can do no harm (source is low), otherwise source is high (+5V)

Constructing Gates

- It turns out that, because the way a transistor works, the easiest gates to create are the NOT, NAND, and NOR gates

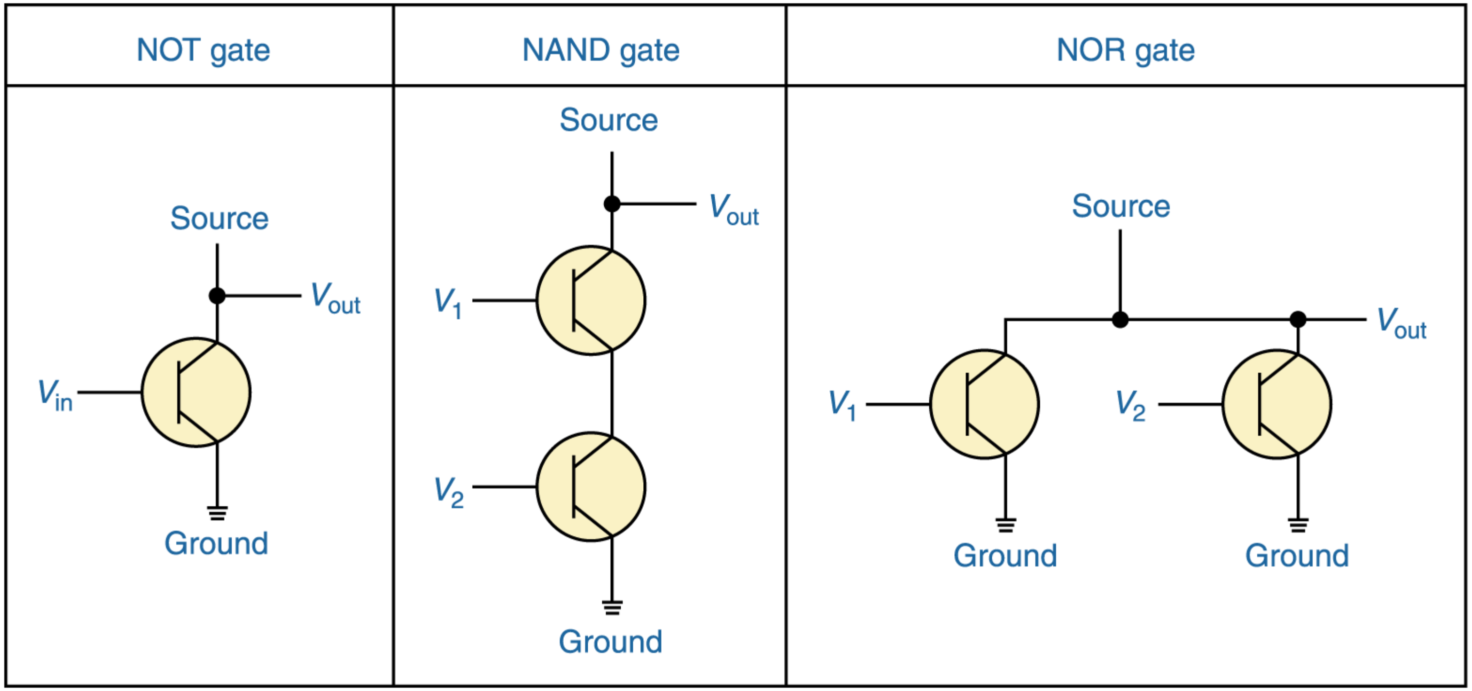


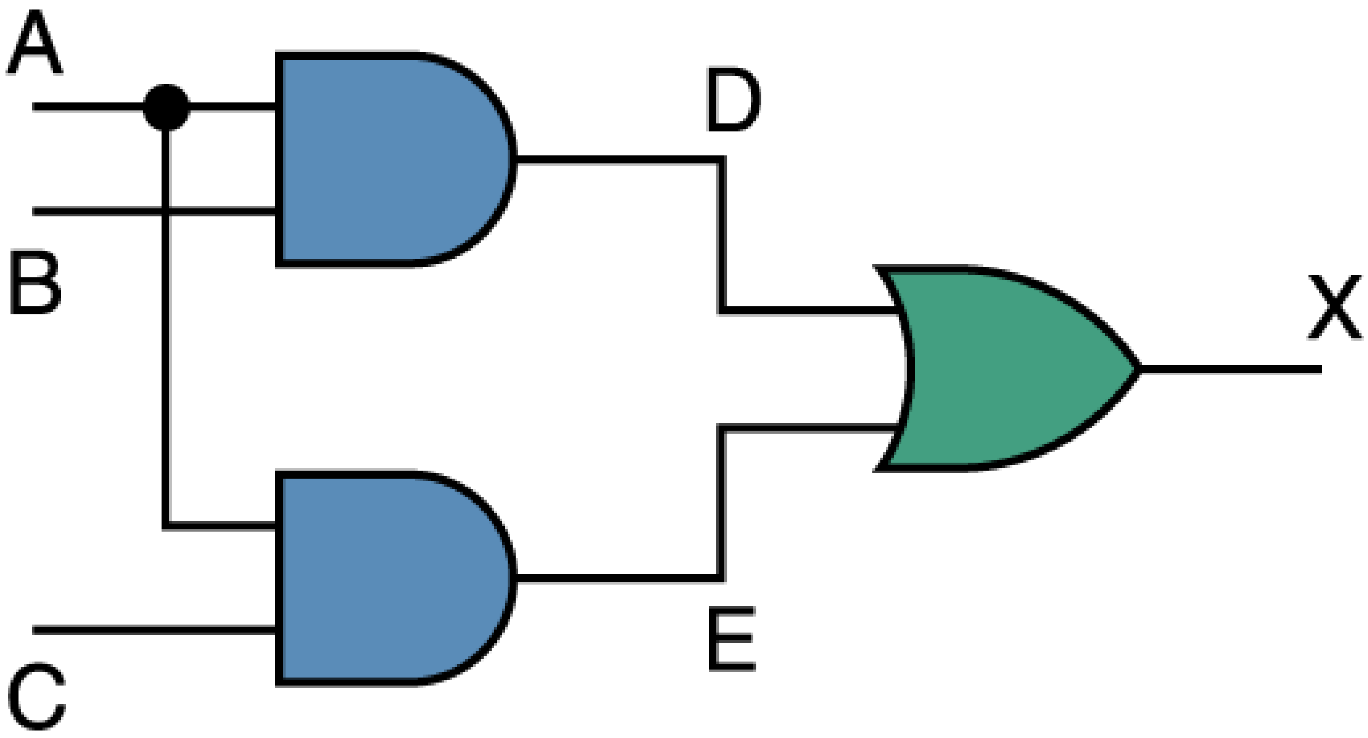
Figure 4.9 **Constructing gates using transistors**

Circuits

- Two general categories
  - In a **combinational circuit**, the input values explicitly determine the output
  - In a **sequential circuit**, the output is a function of the input values as well as the existing state of the circuit
- As with gates, we can describe the operations of entire circuits using three notations
  - Boolean expressions
  - logic diagrams
  - truth tables

Combinational Circuits

- Gates are combined into circuits by using the output of one gate as the input for another



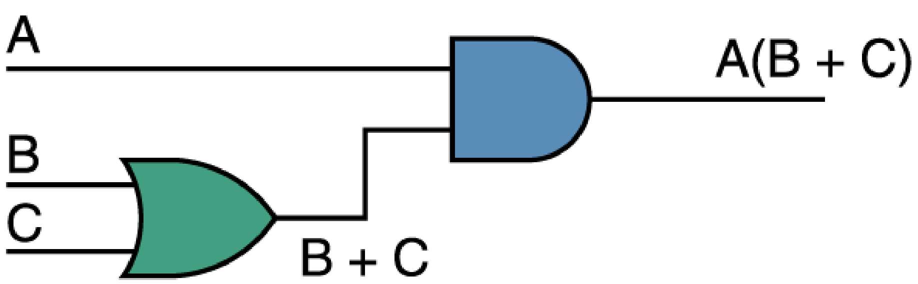
Combinational Circuits

A	B	C	D	E	X
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	0	0	0
1	0	1	0	1	1
1	1	0	1	0	1
1	1	1	1	1	1

- Because there are three inputs to this circuit, eight rows are required to describe all possible input combinations
- This same circuit using Boolean algebra is $(AB + AC)$

Now let's go the other way; let's take a Boolean expression and draw

- Consider the following Boolean expression $A(B + C)$



A	B	C	B + C	A(B+C)
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	1	0
1	0	0	0	0
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

- Now compare the final result column in this truth table to the truth table for the previous example
  - They are identical

Now let's go the other way; let's take a Boolean expression and draw

- We have therefore just demonstrated **circuit equivalence**
  - That is, both circuits produce the exact same output for each input value combination
- Boolean algebra allows us to apply provable mathematical principles to help us design logical circuits

Properties of Boolean Algebra

Property	AND	OR
Commutative	AB = BA	A + B = B + A
Associative	(AB)C = A(BC)	(A + B) + C = A + (B + C)
Distributive	A(B + C) = (AB) + (AC)	A + (BC) = (A + B)(A + C)
Identity	A1 = A	A + 0 = A
Complement	A(A') = 0	A + (A') = 1
DeMorgan's law	(AB)' = A' OR B'	(A + B)' = A'B'

Adders

- At the digital logic level, addition is performed in binary
- Addition operations are carried out by special circuits called, appropriately, **adders**

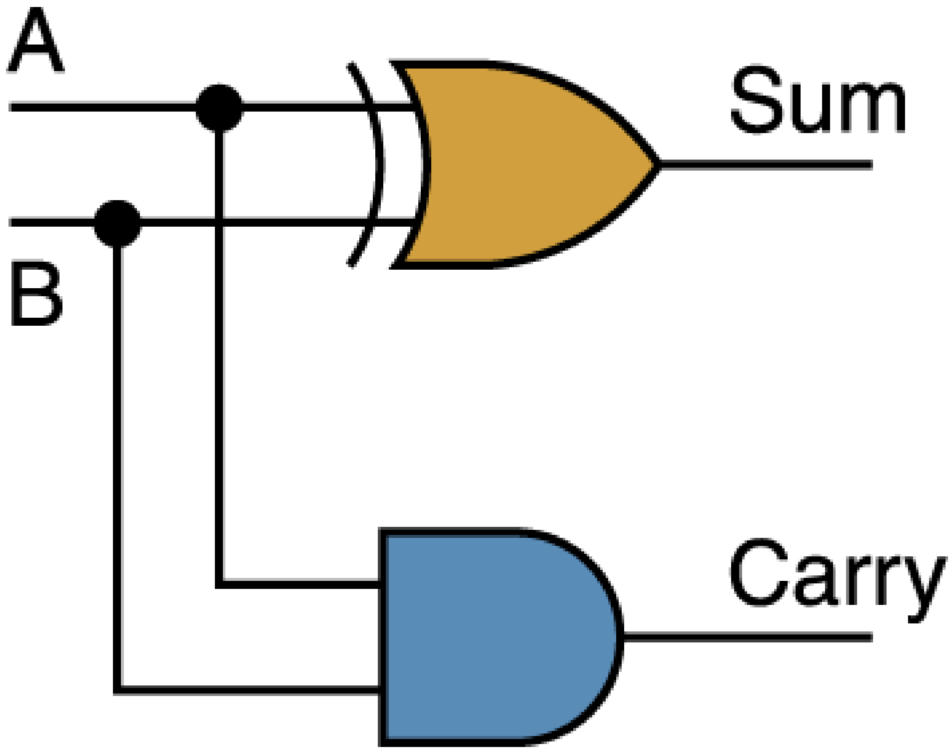
Adders

- The result of adding two binary digits could produce a *carry value*
- Recall that 1 + 1 = 10

  in base two
- A circuit that computes the sum of two bits
  and produces the correct carry bit is called a **half adder**

A	B	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Adders

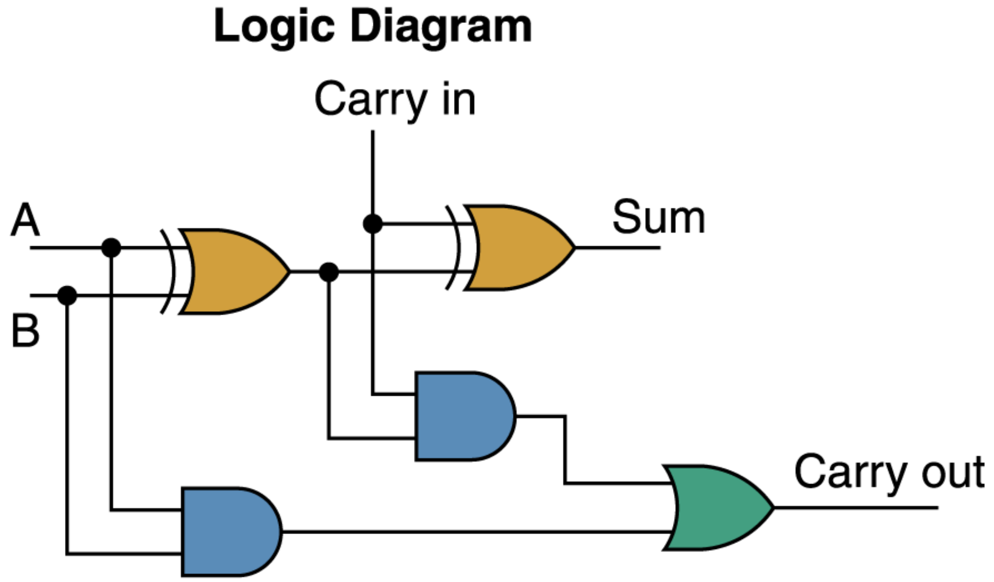


- Circuit diagram representing a half adder
- Two Boolean expressions:

$$\text{sum} = A \oplus B$$
$$\text{carry} = AB$$

Adders

- A circuit called a **full adder** takes the carry-in value into account
- Sum of two binary values with multiple digits each



Truth Table

A	B	Carry-in	Sum	Carry-out
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Multiplexers

- **Multiplexer** is a general circuit that produces a single output signal
  - The output is equal to one of several input signals to the circuit
  - The multiplexer selects which input signal is used as an output signal based on the value represented by a few more input signals, called *select signals* or *select control lines*
  - *animation_telephony_mux_slow.gif*
  - *http://en.wikipedia.org/wiki/Multiplexers*

Multiplexers

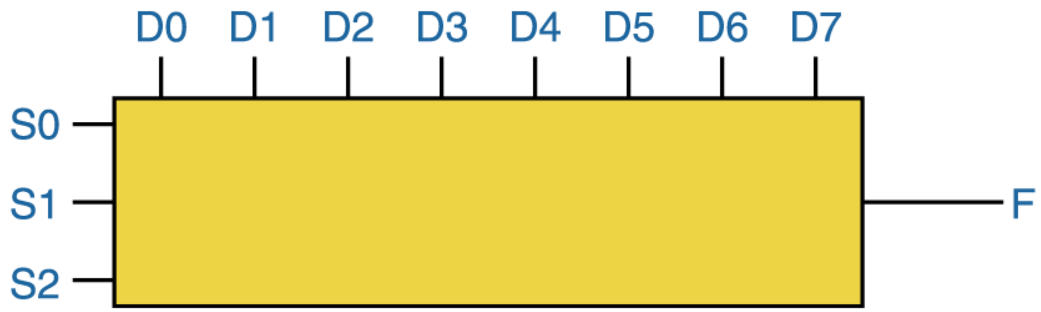


Figure 4.11 A block diagram of a multiplexer with three select control lines

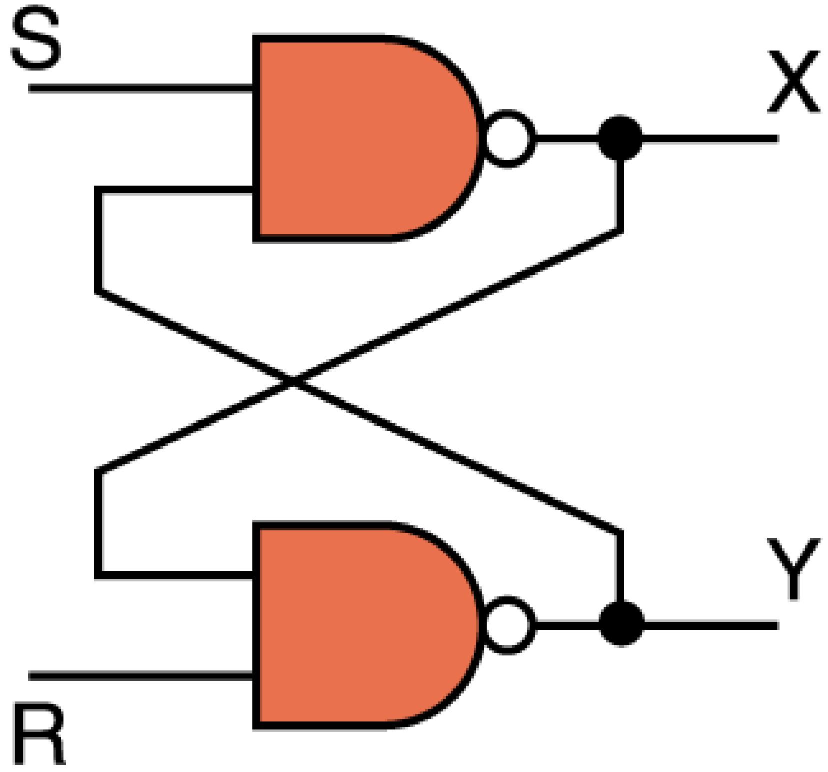
S0	S1	S2	F
0	0	0	D0
0	0	1	D1
0	1	0	D2
0	1	1	D3
1	0	0	D4
1	0	1	D5
1	1	0	D6
1	1	1	D7

- The control lines S0, S1, and S2 determine which of eight other input lines (D0 through D7) are routed to the output (F)

Circuits as Memory

- Digital circuits can be used to store information
- These circuits form a sequential circuit, because the output of the circuit is also used as input to the circuit

Circuits as Memory



- An S-R latch stores a single binary digit (1 or 0)
- There are several ways an S-R latch circuit could be designed using various kinds of gates

- http://en.wikipedia.org/wiki/Flip-flop_%28electronics%29

Circuits as Memory

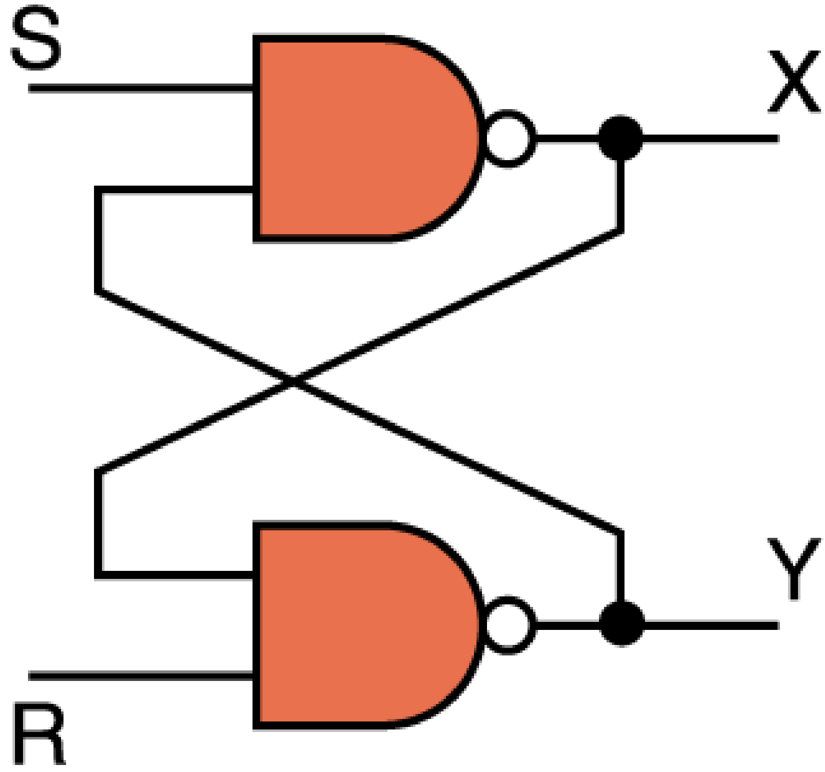
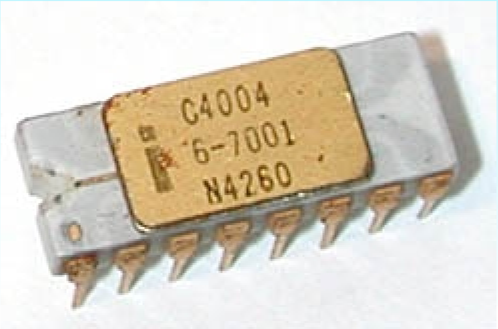


Figure 4.12 **An S-R latch**

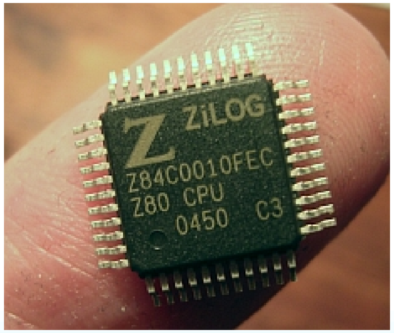
- The design of this circuit guarantees that the two outputs X and Y are always complements of each other
- The value of X at any point in time is considered to be the current state of the circuit
- Therefore, if X is 1, the circuit is storing a 1; if X is 0, the circuit is storing a 0

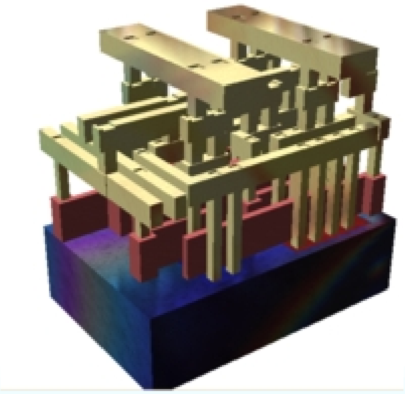
Integrated Circuits



- **Integrated circuit** (also called a *chip*) A piece of silicon on which multiple gates have been embedded

  These silicon pieces are mounted on a plastic or ceramic package with pins along the edges that can be soldered onto circuit boards or inserted into appropriate sockets





Integrated Circuits

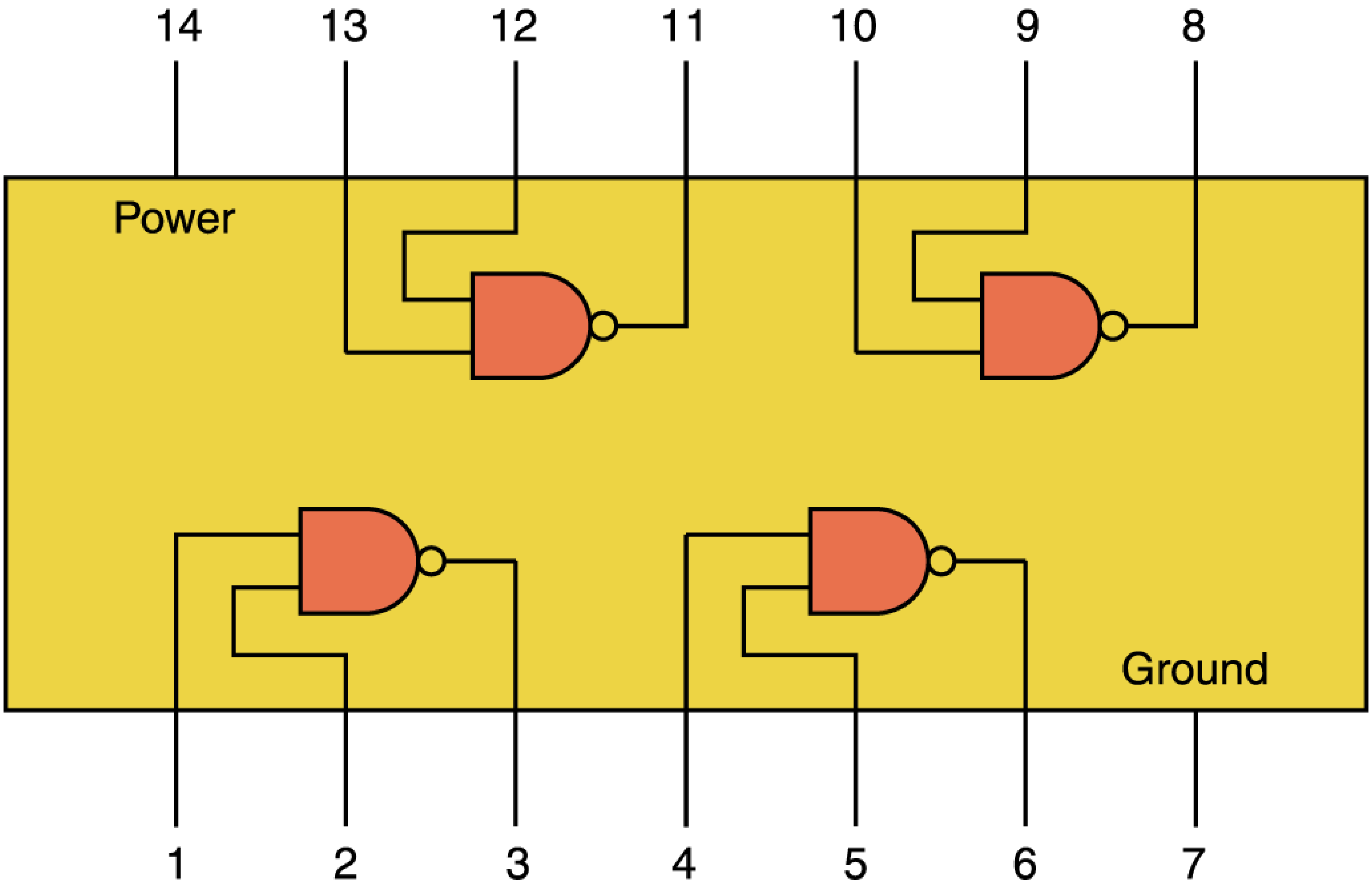
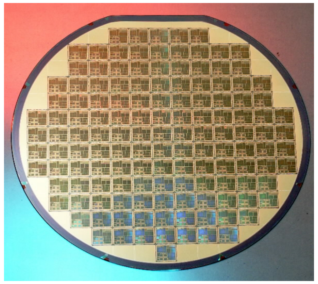


Figure 4.13 An SSI chip contains independent NAND gates

Integrated Circuits

- Integrated circuits (IC) are classified by the number of gates contained in them

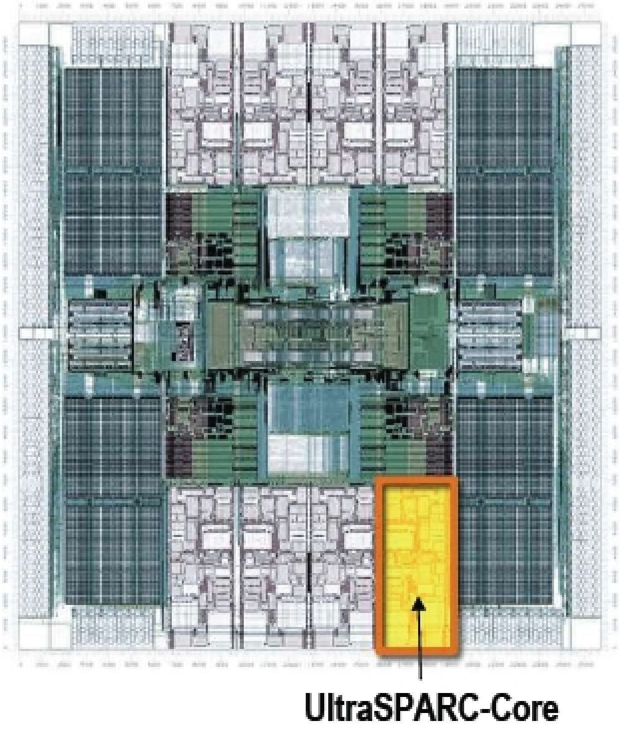
Abbreviation	Name	Number of Gates
SSI	Small-Scale Integration	1 to 10
MSI	Medium-Scale Integration	10 to 100
LSI	Large-Scale Integration	100 to 100,000
VLSI	Very-Large-Scale Integration	more than 100,000



- Wafer Scale Integration

CPU Chips

- The most important integrated circuit in any computer is the Central Processing Unit, or CPU
- Each CPU chip has a large number of pins through which essentially all communication in a computer system occurs



Assignment One

- **Let Me Know If I Can Publish On Web Site**
- **There No Obligation**

Homework

- **Read Chapter Four, Sections 4.4 – 4.7**
- **Exercise: P119-120, 55 & 59**

Mid-Term

- Take Home Exam – Non-Trivial (think!)
- Cover Chapters 1-5 & 16 & Anything Covered In Class
- Given Out: Oct 11
- Due Back: Oct 18
- No Lateness!!!

Good Night