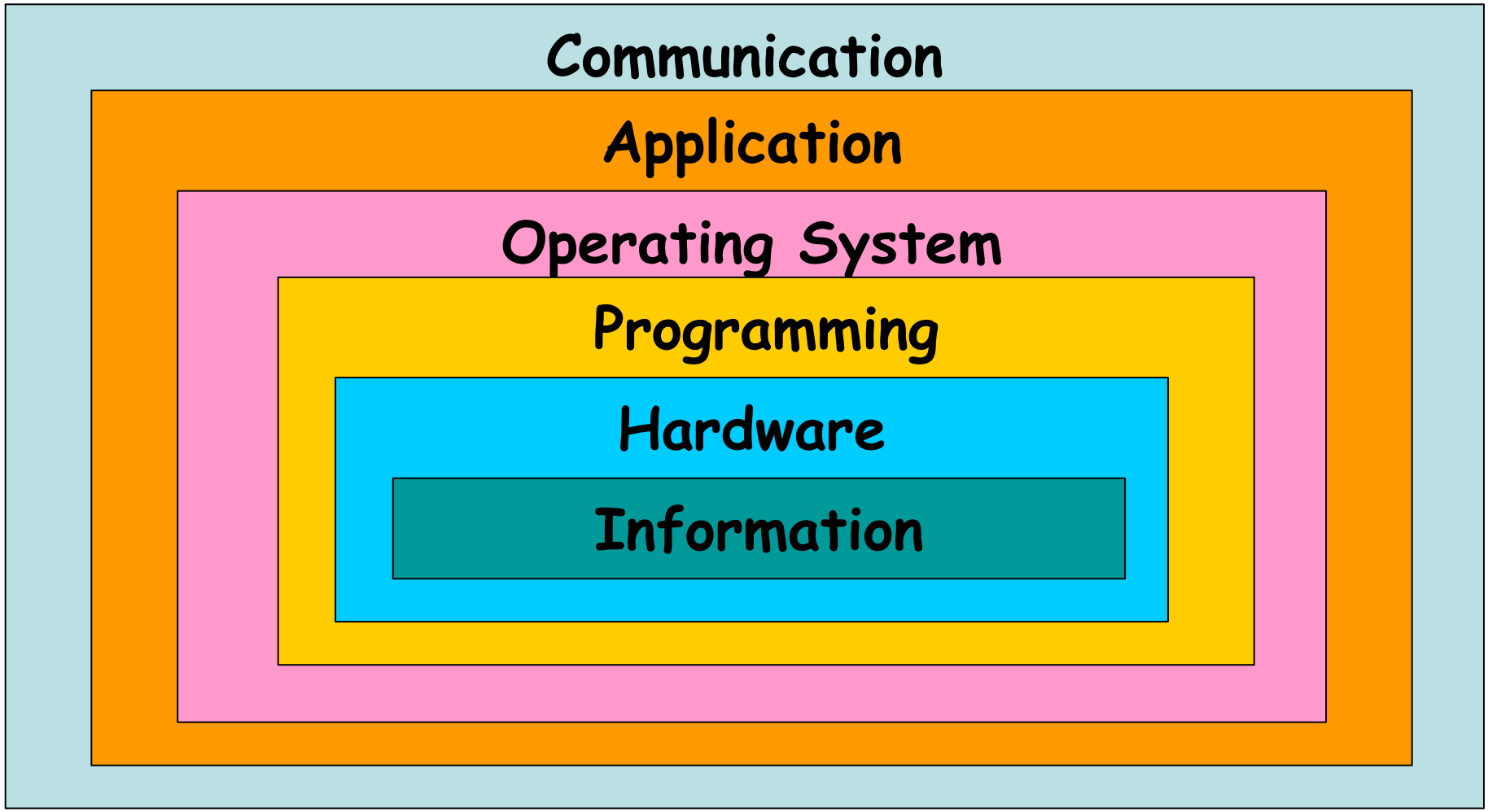
Chapter 2

Binary Values and Number Systems

Layers of a Computing System



Chapter Goals

- Know the different types of numbers
- Describe the relationship between bases 2, 8, and 16
- Conversion between bases
- Why in the world would you ever want to know this?

Numbers

Natural Numbers

Zero and any number obtained by repeatedly adding one to it.

Examples: 100, 0, 45645, 32

Negative Numbers

A value less than 0, with a – sign

Examples: -24, -1, -45645, -32

Numbers

Integers

A natural number, a negative number, zero

Examples: 249, 0, - 45645, - 32

Rational Numbers

An integer or the quotient of two integers

Examples: -249, -1, 0, 3/7, -2/5

Natural Numbers

How many ones are there in 642?

600 + 40 + 2 ?

Or is it

384 + 32 + 2 ?

Or maybe…

1536 + 64 + 2 ?

Natural Numbers

Aha!

642 is 600 + 40 + 2 in **BASE 10**

The **base** of a number determines the number of digits and the value of digit positions

Positional Notation

Continuing with our example…

642 in base 10 *positional notation* is:

$$6 \times 10^2 = 6 \times 100 = 600$$
$$+ 4 \times 10^1 = 4 \times 10 = 40$$
$$+ 2 \times 10^0 = 2 \times 1 = 2 = 642 \text{ in base } 10$$

This number is in base 10

The power indicates the position of the number

Positional Notation

R is the base of the number

As a formula:

$$d_n * R^{n-1} + d_{n-1} * R^{n-2} + \ldots + d_2 * R + d_1$$

n is the number of digits in the number

d is the digit in the i^{th} position in the number

642 is $6_3 * 10^2 + 4_2 * 10 + 2_1$

Positional Notation

What if 642 has the base of 13?

$+ 6 \times 13^2 = 6 \times 169 = 1014$
$+ 4 \times 13^1 = 4 \times 13 = 52$
$+ 2 \times 13^0 = 2 \times 1 = 2$
$= 1068$ in base 10

642 in base 13 is equivalent to 1068 in base 10

Binary

Decimal is base 10 and has 10 digits:

0,1,2,3,4,5,6,7,8,9

Binary is base 2 and has 2 digits:

0,1

For a number to exist in a given number system, the number system must include those digits. For example, the number 284 only exists in base 9 and higher.

Bases Higher than 10

How are digits in bases higher than 10 represented?

With distinct symbols for 10 and above.

Base 16 has 16 digits:

0,1,2,3,4,5,6,7,8,9,A,B,C,D,E, and F

Converting Octal to Decimal

What is the decimal equivalent of the octal number 642?

$$
\begin{aligned}
6 \times 8^2 &= 6 \times 64 &&= 384 \\
+\ 4 \times 8^1 &= 4 \times 8 &&= 32 \\
+\ 2 \times 8^0 &= 2 \times 1 &&= 2 \\
& & &= 418 \text{ in base } 10
\end{aligned}
$$

Converting Hexadecimal to Decimal

What is the decimal equivalent of the hexadecimal number DEF?

$$
\begin{aligned}
D \times 16^2 &= 13 \times 256 = 3328 \\
+\ E \times 16^1 &= 14 \times 16 = 224 \\
+\ F \times 16^0 &= 15 \times 1 = 15 \\
&= 3567 \text{ in base } 10
\end{aligned}
$$

Remember, the digits in base 16 are
0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F

Converting Binary to Decimal

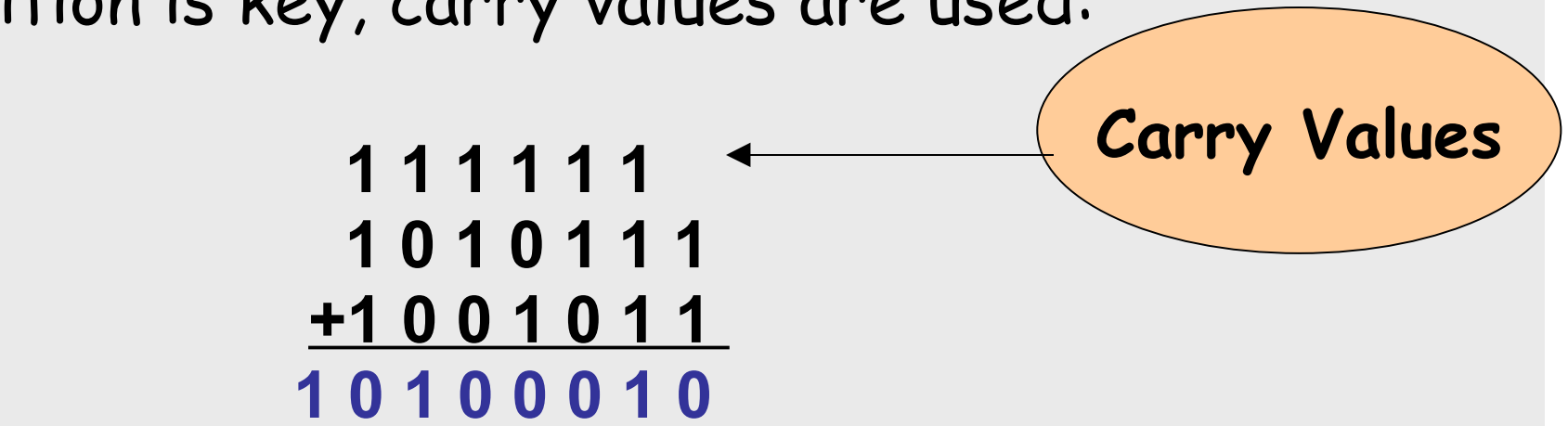
What is the decimal equivalent of the binary number 1101110?

$$
\begin{aligned}
1 \times 2^6 &= 1 \times 64 = 64 \\
+\ 1 \times 2^5 &= 1 \times 32 = 32 \\
+\ 0 \times 2^4 &= 0 \times 16 = 0 \\
+\ 1 \times 2^3 &= 1 \times 8 = 8 \\
+\ 1 \times 2^2 &= 1 \times 4 = 4 \\
+\ 1 \times 2^1 &= 1 \times 2 = 2 \\
+\ 0 \times 2^0 &= 0 \times 1 = 0 \\
&= 110 \text{ in base } 10
\end{aligned}
$$

Arithmetic in Binary

Remember that there are only 2 digits in binary, 0 and 1

Position is key, carry values are used:



Subtracting Binary Numbers

Remember borrowing? Apply that concept here:

```
  1 2
  2 0 2
1 0 1 0 1 1 1
- 1 1 1 0 1 1
-------------
0 0 1 1 1 0 0
```

Power of 2 Number System

Binary	Octal	Decimal
000	0	0
001	1	1
010	2	2
011	3	3
100	4	4
101	5	5
110	6	6
111	7	7
1000	10	8
1001	11	9
1010	12	10

Converting Binary to Octal

- Groups of Three (from right)
- Convert each group

10101011 **10 101 011**

2 5 3

10101011 is 253 in base 8

Converting Binary to Hexadecimal

- Groups of Four (from right)
- Convert each group

10101011 1010 1011

A B

10101011 is AB in base 16

Converting Decimal to Other Bases

Algorithm for converting base 10 to other bases

While the quotient is ***not*** zero:

* Divide the decimal number by the new base

* Make the remainder the next digit to the left in the answer

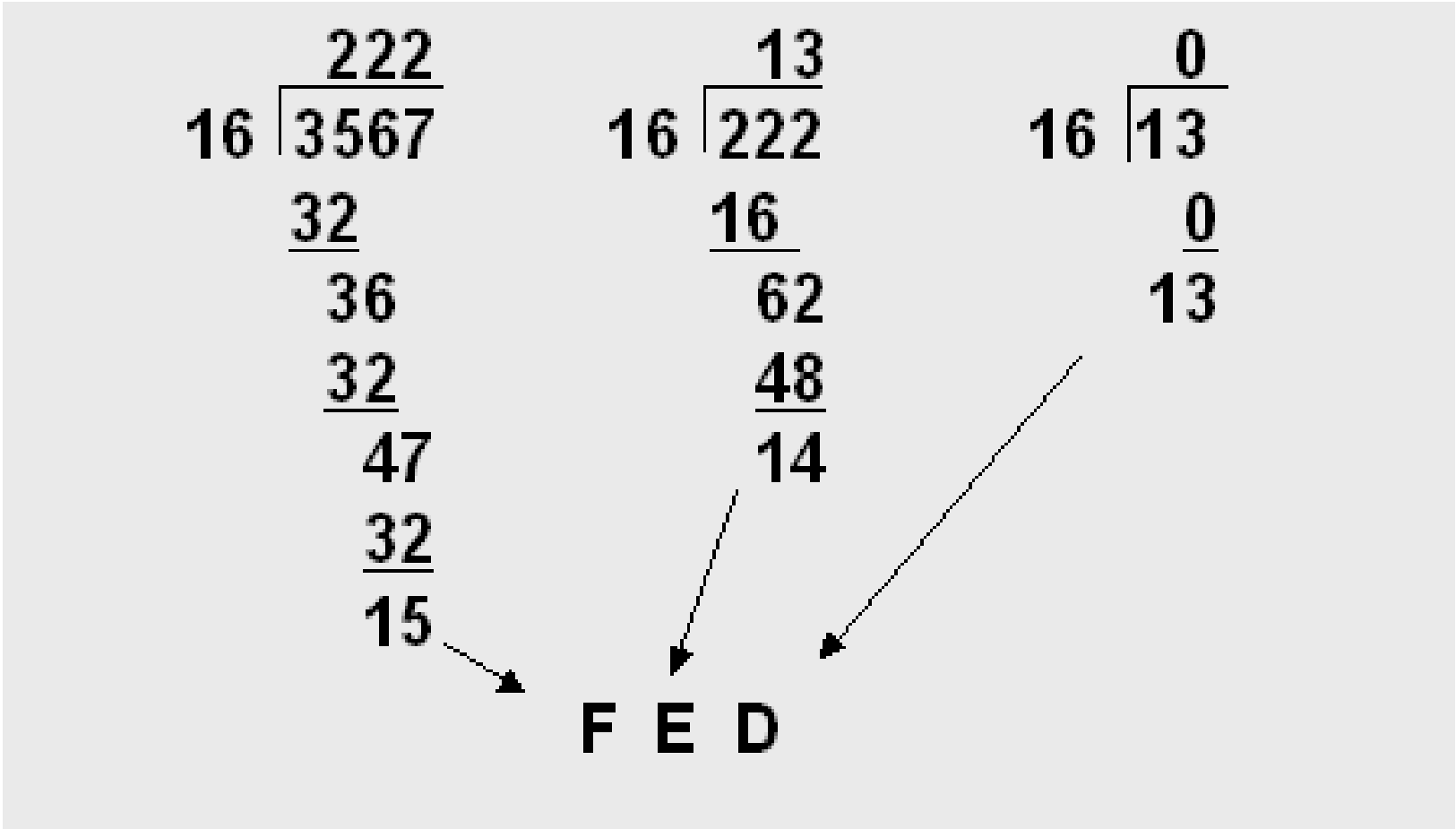
* Replace the original dividend with the quotient

Converting Decimal to Hexadecimal

Try a Conversion

The base 10 number 3567 is what number in base 16?

Converting Decimal to Hexadecimal



Binary and Computers

Binary computers have storage units called binary digits or bits

Low Voltage = 0
High Voltage = 1 all bits have 0 or 1

Binary and Computers

Byte 8 bits

The number of bits in a word determines the word length of the computer, but it is usually a multiple of 8

- 32-bit machines
- 64-bit machines etc.

Converting Binary to Decimal

What is the decimal equivalent of the binary number 1101110?

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\begin{aligned}
1 \times 2^6 &= 1 \times 64 = 64 \\
+\ 1 \times 2^5 &= 1 \times 32 = 32 \\
+\ 0 \times 2^4 &= 0 \times 16 = 0 \\
+\ 1 \times 2^3 &= 1 \times 8 = 8 \\
+\ 1 \times 2^2 &= 1 \times 4 = 4 \\
+\ 1 \times 2^1 &= 1 \times 2 = 2 \\
+\ 0 \times 2^0 &= 0 \times 1 = 0 \\
&= 110 \text{ in base } 10
\end{aligned}
$$

Why, Why Me?

- Why in the world would you ever want to know this?

http://www.nytimes.com

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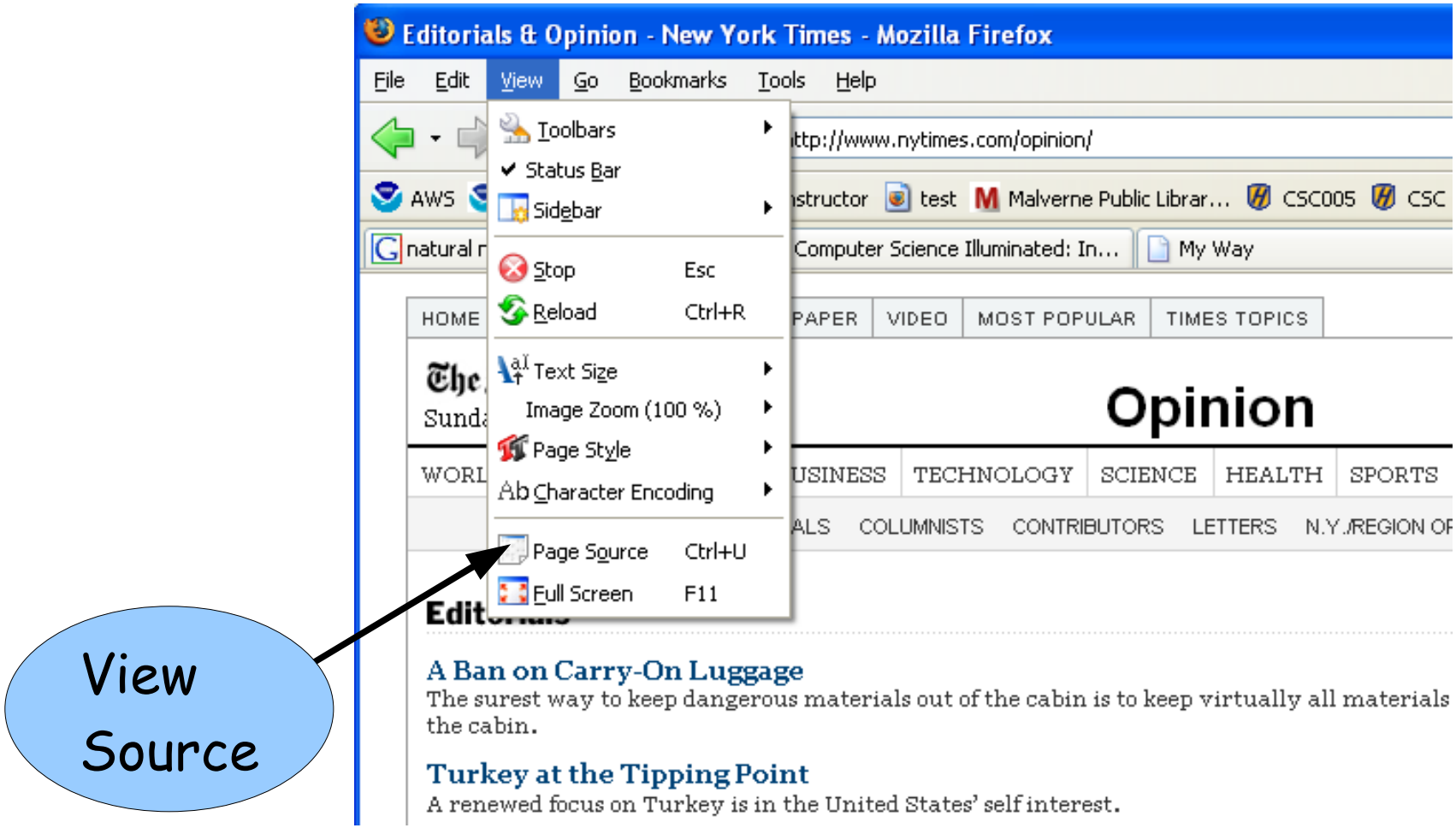

HyperText Markup Language

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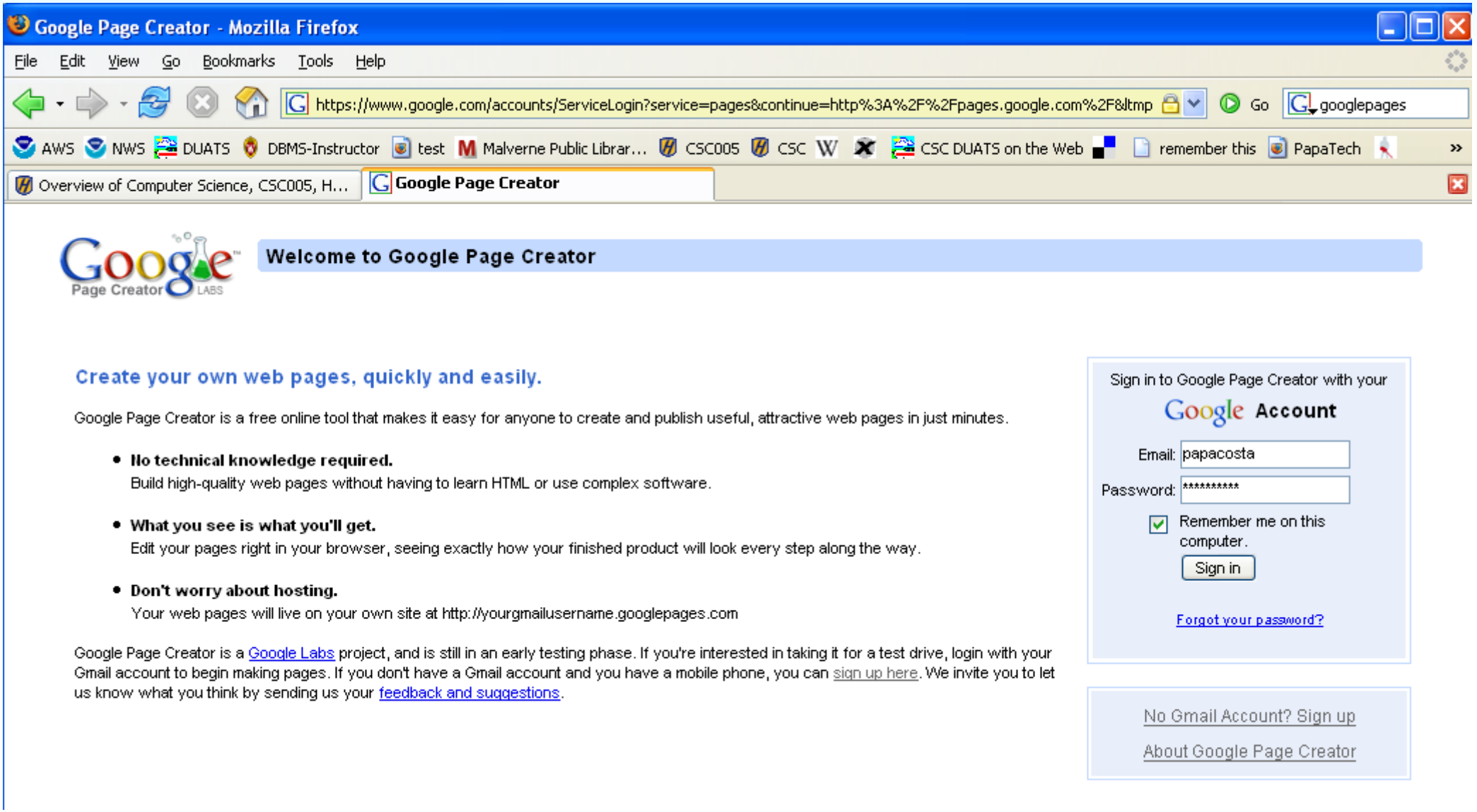
http://pages.google.com



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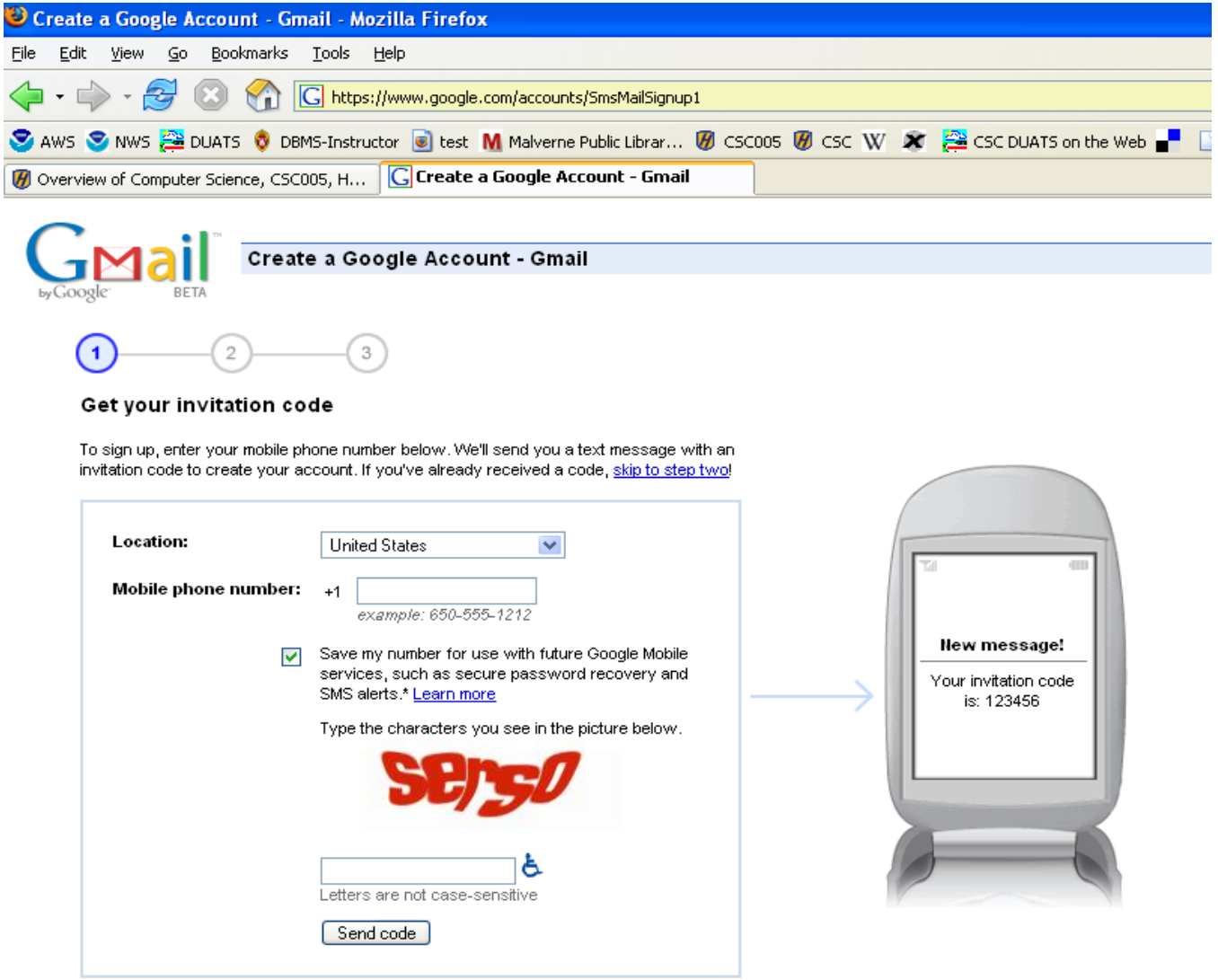


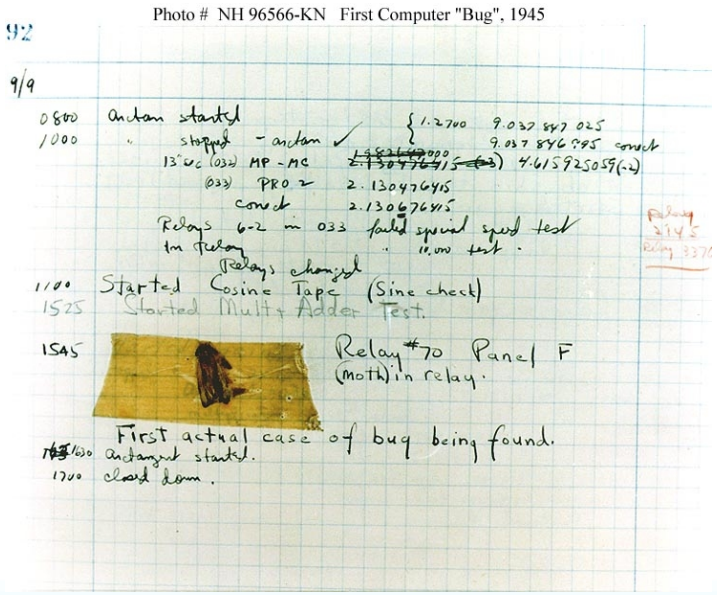
Homework

- Get a gmail account…
- …if you don't want to use your mobile (or you don't have one), send an email to: papacosta@gmail.com
- When you get an account, send me a message!papacosta@gmail.com

The First Compiler…and Bug!

Grace Murray Hopper (December 9, 1906 – January 1, 1992) was an early computer pioneer. She was the first programmer for the Mark I Calculator and the developer of the first compiler for a computer programming language. Hopper was born **Grace Brewster Murray**. She graduated Phi Beta Kappa from Vassar College with a bachelor's degree in mathematics and physics in 1928 and 1934 became the first woman to receive a Ph.D. in mathematics.

She was well-known for her lively and irreverent speaking style, as well as a rich treasury of early "war stories". While she was working on a Mark II computer at Harvard University, her associates discovered a moth stuck in a relay and thereby impeding operation, whereupon she remarked that they were **"debugging"** the system. Though the term **computer bug** cannot be definitively attributed to Admiral Hopper, she did bring the term into popularity. The remains of the moth can be found in the group's log book at the Naval Surface Warfare Center in Dahlgren, VA



http://ei.cs.vt.edu/~history/Hopper.Danis.html

Homework

- Read Chapter Two
- Come Back With Questions

...Have A Nice Night!