CSC185/ENGG185 Fall 2005, Assignment 7 Solutions

1. Page 156/1a

   We are dealing with the standard normal distribution $N(0,1)$, and we are looking for the area under the normal curve on the left of $z = 1.43$, that is $P(Z \leq z) =?$ where $Z \sim N(0,1)$. Directly from the table for the standard normal distribution $N(0,1)$ we find the $P(X \leq 1.43) = 0.9236$

2. Page 156/2b. Find the value of z if the area under the standard normal curve to the left of z is 0.1131. Thus if we think of a random variable $Z \sim N(0,1)$, we are given $P(Z \leq z) = 0.1131$, where z is unknown. To find it we use the table for the standard normal curve in reverse, i.e. find inside the table 0.1131, and track down which is the value of z that corresponds to it. We obtain $z = -1.21$.

3. Page 156/3c. Given $Z \sim N(0,1)$. Find the value of k such that $P(-0.93 < Z < k) = 0.7235$.

   $$0.7235 = P(-0.93 < Z < k) = P(Z \leq k) - P(Z \leq -0.93)$$

   From the table we find $P(Z \leq -0.93) = 0.1762$, thus

   $$0.7235 = P(Z \leq k) - 0.1762$$

   and so

   $$P(Z \leq k) = 0.7235 + 0.1762 = 0.8997$$

   Using the table for the normal distribution in reverse, we find that 0.8997 corresponds to $z = 1.28$. Thus $k = 1.28$.

4. Page 157/4d.

   Given the normal distribution with $\mu = 30$, $\sigma = 6$, find the value of x for which 80% of the area under the normal curve is on the left of x. So if the random variable $X \sim N(30,6)$, we look for x such that $P(X \leq x) = 0.8$.

   We cannot work with the table for the standard normal distribution, first we have to do the Z-transform,

   $$Z = \frac{X - \mu}{\sigma}, \text{ where } Z \sim N(0,1)$$

   and

   $$0.8 = P(X \leq x) = P(Z \leq z)$$

   where

   $$z = \frac{x - \mu}{\sigma}$$

   Now using the table for the standard normal distribution in reverse, we find that for $z = 0.84$, $P(Z \leq z) = 0.8$. Thus

   $$x = z\sigma + \mu = 0.84(6) + 30 = 3.04$$

5. Page 157/7.

   Mice live on avg 40 months.

   X - R.V., the life time of mice, $X \sim N(40, 6.3)$.

   (a) Find $P(X > 32)$.

   First we note that

   $$P(X > 32) = 1 - P(X \leq 32)$$

   We cannot work with the table for the standard normal distribution to find $P(X \leq 32)$, first we have to do the Z-transform,

   $$Z = \frac{X - \mu}{\sigma}, \text{ where } Z \sim N(0,1)$$

   and

   $$P(X \leq 32) = P(Z \leq z), \text{ where } z = \frac{32 - 40}{6.3} = -1.27$$

   From the table for the standard normal distribution, we find that $P(Z \leq -1.27) = 0.1020$, and thus $P(X \leq 32) = 0.1020$, and finally

   $$P(X > 32) = 1 - P(X \leq 32) = 1 - 0.1020 = 0.898$$

(b) Find $P(X < 28)$.
Since X is continuous R.V., $P(X < 28) = P(X \leq 28)$.
Again, we use the Z transform, and then work with the table.

$$\begin{aligned} P(X \leq 28) &= P(\frac{X-40}{6.3} \leq \frac{28-40}{6.3} = \\ &= P(Z \leq -1.90) = 0.0287. \end{aligned}$$

(c) Find $P(37 < X < 49)$.
We use the Z transform, and then work with the table.

$$\begin{aligned} P(37 < X < 49) &= P(\frac{37-40}{6.3} < \frac{X-40}{6.3} < \frac{49-40}{6.3}) = \\ &= P(-0.48 < Z < 1.43) = \\ &= P(Z \leq 1.43) - P(Z \leq -0.48) = \\ &= 0.9236 - 0.3156 = 0.608 \end{aligned}$$

6. Page 157/10.
X - R.V., inside diameter of a piston ring in cm, $X \sim N(10, 0.03)$.

(a) Find $P(X > 10.075)$

$$\begin{aligned} P(X > 10.075) &= 1 - P(X \leq 10.075) = \\ &= 1 - P(\frac{X-10}{0.03} \leq \frac{10.075-10}{0.03}) = \\ &= 1 - P(Z \leq 2.5) = 1 - 0.9938 = 0.0062 \end{aligned}$$

(b) Find $P(9.97 < X < 10.03)$.
One way to solve the problem is to use

$$P(9.97 < X < 10.03) = P(X \leq 10.03) - P(X \leq 9.97)$$

and solve exactly as the "mice" problem, part c.
Another way, is to notice that the interval $[9.97, 10.03]$ is symmetric around the mean 10 of the normal distribution, thus $P(X \leq 9.97) = P(X \geq 10.03)$, and we use

$$\begin{aligned} P(9.97 < X < 10.03) &= 1 - (P(X \leq 9.97) + P(X \geq 10.03)) = \\ &= 1 - 2P(X \leq 9.97) = \\ &= 1 - 2P(\frac{X-10}{0.03} \leq \frac{9.97-10}{0.03}) = \\ &= 1 - 2P(Z \leq -1) = 1 - 2(0.1587) = 0.6826 \end{aligned}$$

(c) Find x such that $P(X < x) = 0.15$.

$$0.15 = P(X < x) = P(\frac{X-10}{0.03} < \frac{x-10}{0.03} = P(Z \leq z)$$

From the table for the standard norma;l distribution, we find $z = -1.04$. Thus

$$x = (0.03)z + 10 = (0.03)(-1.04) + 10 = 9.9688$$

7. Page 158/21 X - R.V., daily amount of coffee in litters, X has uniform distribution, $X \sim \mathcal{U}(7, 10)$.
The pdf of X is

$$f(x) = \frac{1}{3}, \ 7 \leq x \leq 10; \text{ and 0 otherwise}$$

(a) Find $P(X \leq 8.8)$.

$$P(X \leq 8.8) = \int_7^{8.8} (1/3)dx = (1/3)x|_7^{8.8} = (8.8 - 7)/3 = 0.6$$

(b) Find $P(7.4 < X < 9.5)$.
You could find the answer by integrating the pdf between 7.4 and 9.5, as in part a).
But you could also notice that the area under the pdf between 7.4 and 9.4 is actualy the are of a rectangle with sides 1/3 and $9.5 - 7.4 = 2.1$, and the area of such a rectangle is $2.1/3 = 0.7$.

(c) Find $P(X \geq 8.5)$.
Again, either integrate the pdf from 8.5 to 10, or find the area of a rectangle with sides 1/3 and $10 - 8.5 = 1.5$. The area of the rectangle is $1.5/3 = 0.5$.

8. Page 158/22.

Bus arrives every 10min at a bus stop.

X- R.V., the waiting time for a bus, X has uniform distribution,$X \sim \mathcal{U}(0, 10)$.

The pdf of X is

$$f(x) = \frac{1}{10}, \ 0 \leq x \leq 10; \text{ and 0 otherwise}$$

(a) Find $P(X > 7)$.
Again, either integrate the pdf from 7 to 10, or find the area of a rectangle with sides 1/10 and $10 - 7 = 3$. The area of the rectangle is $3/10 = 0.3$.

(b) Find $P(2 < X < 7)$.
Again, either integrate the pdf from 2 to 7, or find the area of a rectangle with sides 1/10 and $7 - 2 = 5$. The area of the rectangle is $5/10 = 0$.

9. Page 124/4

75% of all thefts are for drugs.

X - R.V., the number of thefts for drugs out of 5 thefts total.

The probability that a theft is for drugs is $0.75 = 3/4$.

So we have a sequence of 5 Bernoulli trials with probability of success (i.e. drug related theft) 3/4, and X has Binomial distribution, $X \sim B(5, 3/4)$, $n = 5, p = 3/4$.

(a) Find $P(X = 2)$.
We use the definition of the Binomial pmf substituting $n = 5, p = (3/4), x = 2$

$$F(X = 2) = C_2^5 (3/4)^2 (1 - 3/4)^{5-2} = \frac{5!}{2!(5-2)!}(3/4)^2(1/4)^3 = 0.0879$$

(b) Find $P(X \leq 3)$.
You can find this either by summing the pmf for $x = 0, 1, 2, 3$, or you can look up this value directly form the table for the Binomial distribution with parameters $n = 5, p = 0.75$
$P(X \leq 3) = 0.3672$.

10. Page 125/7.

70% of lang cancer patients are chain smokers.

(a) X - R.V., number of chain smokers among 10 cancer patients.
We have $n = 10$ Bernoulli trials with a probability of success (chain smoker)$p = 0.7$. X has Binomial distribution,$X \sim B(10, 0.7)$.
Find $P(X < 5)$.
Note that in the table for the Binomial distribution we have only probabilities $P \leq r$, thus we have to rework the above expression first and then look it up in the table.

$$P(X < 5) = P(X \leq 4) = 0.0474$$

(b) X - R.V., number of chain smokers among 20 cancer patients.

We have $n = 20$ Bernoulli trials with a probability of success (chain smoker)$p = 0.7$. X has Binomial distribution,$X \sim B(20, 0.7)$.

Find $P(X < 10)$.

Note that in the table for the Binomial distribution we have only probabilities $P \leq r$, thus we have to rework the above expression first and then look it up in the table.

$$P(X < 10) = P(X \leq 9) = 0.0171$$